### REPORT DOCUMENTATION PAGE

collection of information, increasing suggestions for reducing this burden, to Washington Headequarte Ozers Highway, Suize 1204, Arkington, VA. 22202-4302, and to the Office of Management and Budger,

Public resorting bursen for this collection of information is estimated to average I hour Set response, including suffering and maintaining the data needed, and completing and reviewing the collection of information. See

AFRL-SR-BL-TR-98-

1. AGENCY USE ONLY (Leave blank) 2. REPORT DATE 3. REPORT TYPE AND DATES COVER Final Tech/11/01/96-10 Feb 1998 A TITLE AND SUBTITLE 5. FUNDING NUMBERS Non-Fickian Diffusive Transport in Modern F49620-94-1-0044 Polymeric Materials. & AUTHOR(S) Donald S. Cohen, Principal Investigator 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) E. PERFORMING ORGANIZATION REPORT NUMBER Applied Mathematics, 217-50 California Institute of Technology Pasadena, CA 91125 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) 10. SPONSORING/MONITORING AGENCY REPORT NUMBER Administrative Contracting Office AFOSR/PKA 110 Duncan Avenue, Suite B115 Bolling AFB, D.C. 20332-0001 11. SUPPLEMENTARY NOTES 122. DISTRIBUTION/AVAILABILITY STATEMENT 12b. DISTRIBUTION CODE Approved for public release; distribution unlimited. 13. ABSTRACT (Maximum 200 words) We have developed a model for the theoretical description of large classes of problems involving diffusive transport and mechanical relaxation in polymers undergoing glass-rubber transitions. The derivation and details of the model have been described in previous progress reports. In the final year of AFOSR support we proceeded simultaneously in two directions, namely to formulate and solve moving boundary problems for the penetrant fronts and to investigate the implications of the 'forbidden shock' regions for diffusive systems.

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14. SUBJECT TERMS			15. NUMBER OF PAGES
diffusive transport, mechanical relaxation, polymer penetrant problems, non-Fickian diffusion.			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION :OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UNLIMITED

# Final Technical Report AFOSR Grant F49620-94-1-0044 1 November 1996 through 31 October 1997

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We have developed a model for the theoretical description of large classes of problems involving diffusive transport and mechanical relaxation in polymers undergoing glass-rubber transitions. The derivation and details of the model have been described in previous progress reports. In the final year of AFOSR support we proceeded simultaneously in two directions.

- (1) Particularly interesting and formidable were the new classes of moving boundary problems needed to describe the evolution of the penetrant front. Due to the inherent multiple time and space scales involved, similarity methods could not be used, and multi-scale techniques were devised for the full partial differential equations involved. With experience gained from developing our methods on several early simplified problems we have now successfully done physically realistic problems for both Case II and Super-Case II diffusion into a glassy polymer. We have isolated the parameter dependencies and controlling factors for the propagating diffusive fronts.
- (2) Certain polymer-penetrant problems give rise to unusual nonlinear, non-Fickian diffusion alone or in combination with mechanical relaxation and/or reaction. The unusual nature of these new problems comes from the form of the conditions at fixed and moving boundaries. Preliminary results obtained by T. P. Witelski in his thesis research for D. S. Cohen indicated that evolution equations with interesting time dependent forcing account for the propagation of sharp interfaces and the formation of shocks. This time dependence is introduced from the original boundary conditions even when there is no time dependent forcing in the original equations. We have pursued this and studied the process by which the time dependence causes subtle changes in the shock formation process, including the creation of "forbidden regions" where shocks are expected from the more usual studies of reaction-diffusion equations subject to standard mathematical boundary conditions, but where they can not form in the present problems. This will have serious implications with regard to the fabrication and design of many polymeric materials.

#### **Publications**

- D. A. Edwards, Constant Front Speed in Weakly Non-Fickian Diffusive Systems. SIAM J. Appl. Math., 55 (1995) 1039-1058.
- D. S. Cohen, C.J. Durning and D.A. Edwards, Perturbation analysis of Thomas and Windle's model of Case II transport, AIChE Journal, 42 (1996) 2025-2035.
- D. S. Cohen and T. P. Witelski, Inaccessible states in time-dependent reaction diffusion, Studies in Applied Math., 97 (1996) 301-319.
- D.A. Edwards and D. S. Cohen, The effect of a changing diffusion coefficient in super-case II polymer-penetrant systems." IMA J. of Applied Math.
- D. A. Edwards and D. S. Cohen, A mathematical model for a dissolving polymer, AIChE Journal, <u>41</u>(1995) 2345-2355.
- T. P. Witelski and D. S. Cohen, Forbidden regions for shock formation in diffusive systems, Studies in Applied Math., <u>95</u> (1995) 297-317.
- D. S. Cohen and T. P. Witelski, Perturbed reversible systems, Phys. Letters A, 207 (1995) 83-86.
- D. S. Cohen and T. Erneux, Asymptotic limits for controlled drug release, SIAM J. Appl. Math, to appear.

# CONTRIBUTED PAPER (oral presentation)

S. Xiong, C. Durning, D.S. Cohen, "Swelling and Collapse of Elastic Shells," paper 7i, 1996 Spring National Meeting of the AIChE, New Orleans LA, Feb. 1996.

# CONTRIBUTED PAPER (oral presentation)

C. Durning, S. Xiong, D.S. Cohen and D.A. Edwards, "Swelling and Collapse of Elastic Shells," paper 1 session FM-E, XIIth International Congress on Rheology, Quebec CA, August 1996.

#### PUBLISHED EXTENDED ABSTRACT

C.J. Durning, S. Xiong, D.S. Cohen and D.A. Edwards, "Swelling and Collapse of Elastic Shells," Proc. XIIth Int. Congress on Rheology, A. Ait-Kadi, J.M. Dealy, D.F. James and M.C. Williams eds., Laval University Press, Quebec CA (1996), p.343.

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